Steady-State Analysis of Power Transformers under DC Bias by the Finite Element Method with the Fixed Point Technique

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Abstract — A method using a fixed point technique is proposed for solving steady-state power transformer problems with the voltages of the windings and a DC bias in their current given. Amending the T, Φ-Φ formulation with an additional voltage equation, the fixed point approach enables us to solve the problem in the discrete Fourier domain parallel at each transformed time step. The presented method is illustrated by a numerical example.

I. Introduction

Geomagnetically induced currents (GIC) are direct currents that enter and leave the directly earthed neutrals of high-voltage star connected windings, causing a DC bias in the magnetizing current of the transformer. The frequency of GIC ranges typically from 0.001 Hz to 0.01 Hz, and the peak value could be up to 200 A [1]. The core is saturated during the half cycle in which the bias current is in the same direction as the magnetizing current, causing undesirable effects like increased noise, additional core losses as well as eddy current losses due to the higher leakage flux.

An early analytical method allowing the transformer flux offset and exciting-current waveform to be determined for combined AC and GIC excitation is described in [2]. Magnetic circuit models based on two dimensional finite element techniques have been used in [3]. An experimental analysis has been given in [4]. A method based on three dimensional FEM to determine the waveform of the magnetizing current in a single phase transformer has been presented in [5]. In such investigations, the excitation is the time dependent current density in the transformer windings determined by the time function of the magnetizing current. A method using the voltage as excitation in conjunction with a current vector potential description has been proposed [6], without attempting to consider the DC bias. The problem is solved by a harmonic balance technique using a block Gauss-Seidel technique to separate the harmonics in [7].

This paper focuses on solving nonlinear steady-state power transformer problems under DC bias in the discrete Fourier domain. The voltages in the windings are directly used as the excitation. Applying the fixed point technique [8], there is no coupling between the transformed time steps within a nonlinear iteration and, hence, in contrast to the method of [7], they can be computed parallel.

II. Finite element formulation

Applying the **T**,Φ-Φ formulation to the eddy current problem, the following partial differential equations have to be solved:

$$\nabla\times\rho\nabla\times\mathbf{T}+\frac{\partial}{\partial t}(\mu\mathbf{T}-\mu\nabla\Phi)=-\nabla\times\rho\nabla\times\mathbf{T}_0-\frac{\partial}{\partial t}(\mu\mathbf{T}_0) \quad (1)$$

$$\nabla\cdot(\mu\mathbf{T}-\mu\nabla\Phi)=-\nabla\cdot(\mu\mathbf{T}_0) \quad (2)$$

where ρ, μ, $\mathbf{T}$, $\mathbf{T}_0$, and Φ are the resistivity, the permeability, the reduced electric vector potential, the impressed electric vector potential and the magnetic scalar potential, respectively.

The impressed vector $\mathbf{T}_0$ can be written as $i\mathbf{t}_0$ where i is the current of the coil. When the coil is driven by a voltage source, the current i is unknown and an additional voltage equation is needed [6]. Applying the Galerkin method to (1) as well as to the time derivative of (2) and supplementing the result by the additional voltage equation, one obtains

$$\left(\begin{bmatrix}0&0&0\\0&\mathbf{S}(\rho)&0\\0&0&0\end{bmatrix}+\frac{\partial}{\partial t}\begin{bmatrix}\mathbf{V}(\mu)&\mathbf{g}(\mu)&\mathbf{h}(\mu)\\\mathbf{g}^{\mathrm{T}}(\mu)&\mathbf{M}(\mu)&\mathbf{G}(\mu)\\\mathbf{h}^{\mathrm{T}}(\mu)&\mathbf{G}^{\mathrm{T}}(\mu)&\mathbf{L}(\mu)\end{bmatrix}\right)\begin{bmatrix}i\\\mathbf{T}_h\\\phi_h\end{bmatrix}=\begin{bmatrix}u\\0\\0\end{bmatrix} \quad (3)$$

where **S** is the stiffness matrix depending on the resistivity, and the finite element matrices **M**, **G**, $\mathbf{G}^{\mathrm{T}}$ and **L** depending on the permeability are the mass matrix, the gradient matrix, the divergence matrix and the discrete Laplace matrix, respectively [8]. The matrices **V**, **g** and **h** correspond to $\mathbf{t}_0\mathbf{t}_0\mu$, $\mathbf{t}_0\mu$ and $-\nabla\cdot\mathbf{t}_0\mu$, respectively [6]. The subscript h denotes the finite element approximations, i is the current of the coil, and u is the given voltage.

III. Fixed Point Method

Applying the fixed point method to linearize (3), the following iterative algorithm is obtained:

$$\left(\begin{bmatrix}0&0&0\\0&\mathbf{S}(\rho)&0\\0&0&0\end{bmatrix}+\frac{\partial}{\partial t}\begin{bmatrix}\mathbf{V}\left(\mu_{FP}^{(s)}\right)&\mathbf{g}\left(\mu_{FP}^{(s)}\right)&\mathbf{h}\left(\mu_{FP}^{(s)}\right)\\\mathbf{g}^{\mathrm{T}}\left(\mu_{FP}^{(s)}\right)&\mathbf{M}\left(\mu_{FP}^{(s)}\right)&\mathbf{G}\left(\mu_{FP}^{(s)}\right)\\\mathbf{h}^{\mathrm{T}}\left(\mu_{FP}^{(s)}\right)&\mathbf{G}^{\mathrm{T}}\left(\mu_{FP}^{(s)}\right)&\mathbf{L}\left(\mu_{FP}^{(s)}\right)\end{bmatrix}\right)\begin{bmatrix}i^{(s+1)}\\\mathbf{T}_h^{(s+1)}\\\Phi_h^{(s+1)}\end{bmatrix}$$
$$=\begin{bmatrix}u\\0\\0\end{bmatrix}+\frac{\partial}{\partial t}\begin{bmatrix}\mathbf{V}\left(\mu_{FP}^{(s)}-\mu\right)&\mathbf{g}\left(\mu_{FP}^{(s)}-\mu\right)&\mathbf{h}\left(\mu_{FP}^{(s)}-\mu\right)\\\mathbf{g}^{\mathrm{T}}\left(\mu_{FP}^{(s)}-\mu\right)&\mathbf{M}\left(\mu_{FP}^{(s)}-\mu\right)&\mathbf{G}\left(\mu_{FP}^{(s)}-\mu\right)\\\mathbf{h}^{\mathrm{T}}\left(\mu_{FP}^{(s)}-\mu\right)&\mathbf{G}^{\mathrm{T}}\left(\mu_{FP}^{(s)}-\mu\right)&\mathbf{L}\left(\mu_{FP}^{(s)}-\mu\right)\end{bmatrix}\begin{bmatrix}i^{(s+1)}\\\mathbf{T}_h^{(s+1)}\\\Phi_h^{(s+1)}\end{bmatrix} \quad (4)$$

where $i^{(s+1)}$, $\mathbf{T}_h^{(s+1)}$ and $\Phi_h^{(s+1)}$ are the unknowns in s-th iteration, the permeability $\mu^{(s)}$ is calculated from the s-th solution and the fixed point parameter $\mu^{(s)}{}_{FP}$ is determined by $\mu^{(s)}$ as presented in [8].

To obtain the periodic solution in the time domain, an equidistant time discretization within one period ($\Delta t=T/n$) and a finite difference scheme are used for the time derivative. The equations of the **T**,Φ-Φ formulation with the fixed point method associated with second and third rows of (4) are first solved after discrete Fourier transformation using the value of the current from the previous nonlinear iteration. All discrete Fourier components are independent, so they can be computed parallel [8].

For the 0-th discrete harmonic, i.e. the DC-components, the first and third rows of (4) vanish. According to the second row of (4), $\mathbf{T}_{h,0}^{(s+1)}$ is equal to 0. Since the DC component of the current is known, the discretization of (2) (i.e. the third row of (4) without the time derivative) can be solved for the 0-th discrete harmonic:

$$\mathbf{L}\left(\mu_{FP}^{(s)}\right)\hat{\Phi}_0^{(s+1)} = \mathbf{h}^{\mathrm{T}}\left(\mu_{FP}^{(s)}\right)\hat{i}_0^{(s)} + \mathbb{DF}_k\left(\mathbf{L}\left(\mu_{FP}^{(s)} - \mu\right)\Phi_h^{(s)}\right) \quad (5)$$

where $\mathbb{DF}$ denotes the discrete Fourier transform and $\Phi_h^{(0)}$ in the initial iteration is zero.

Having computed $\mathbf{T}$, Φ and μ at the s-th iteration, the calculation of the current is carried out separately. Applying the fixed point method to the voltage equation yields

$$\begin{aligned}\alpha_k \mathbf{V}\left(\mu_{FP}^{(s)}\right)\hat{i}_k^{(s+1)} &= \hat{u}_k + \alpha_k \mathbb{DF}_k\left(\left[\mathbf{V}\left(\mu_{FP}^{(s)} - \mu\right)\right] i_k^{(s)}\right) \\ &\quad - \alpha_k \mathbb{DF}_k\left(\left[\mathbf{g}(\mu) \quad \mathbf{h}(\mu)\right]\begin{bmatrix}\mathbf{T}_h^{(s)} \\ \phi_h^{(s)}\end{bmatrix}\right)\end{aligned} \quad (6)$$

where α_k is the coefficient corresponding to the chosen discrete difference scheme. The k-th component of the current in the discrete Fourier transform is given by $\mathbb{DF}_k i := \hat{i}_k = \frac{1}{n}\sum_{l=0}^{n-1} i_l e^{-j2\pi kl/n}$ where $i_l = i(l\Delta t)$ is the value of the current at the l-th time step.

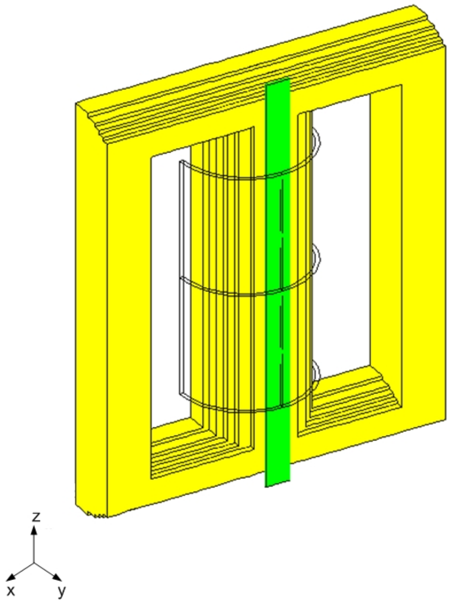

Fig.1 . Model of power transformer with a three-limb core

IV. Numerical results

The geometry of a three-limb core power transformer is shown in Fig.1, including the core, a winding with the magnetizing current and a tie bar carrying eddy currents. The model comprises 54 144 second-order hexahedral finite elements. The ferromagnetic materials of the core and of the tie bar are nonlinear. The winding is excited by a given sinusoidal voltage.

The fixed point method with the time periodic technique uses 40 time steps. The waveform of the winding current at a DC component of 45A obtained by the present method is in good agreement with the result of [5] as shown in Fig.2.

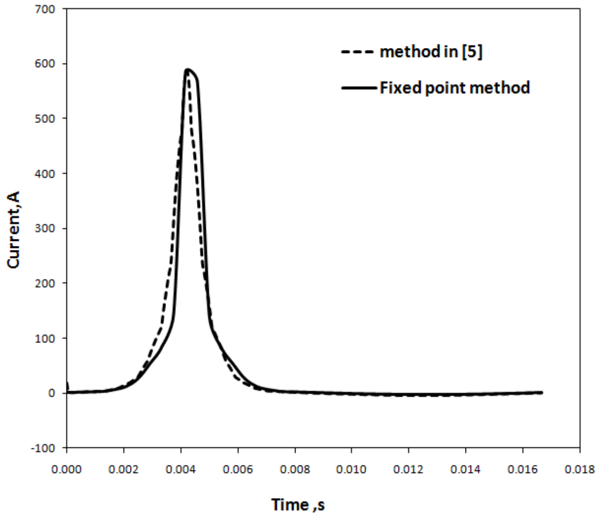

Fig. 2. Comparison of magnetizing current waveforms at DC bias of 45 A

V. Conclusions

We have presented a fixed pointed method to treat voltage driven power transformers under DC bias using the **T**,Φ-Φ formulation and an additional voltage equation. The finite element simulation of the power transformer under DC bias could be carried out without the prediction of the magnetizing current waveform of the windings and the steady state solution is obtained without stepping through transients. The computation of the discrete Fourier components can be carried out parallel.

VI. References

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